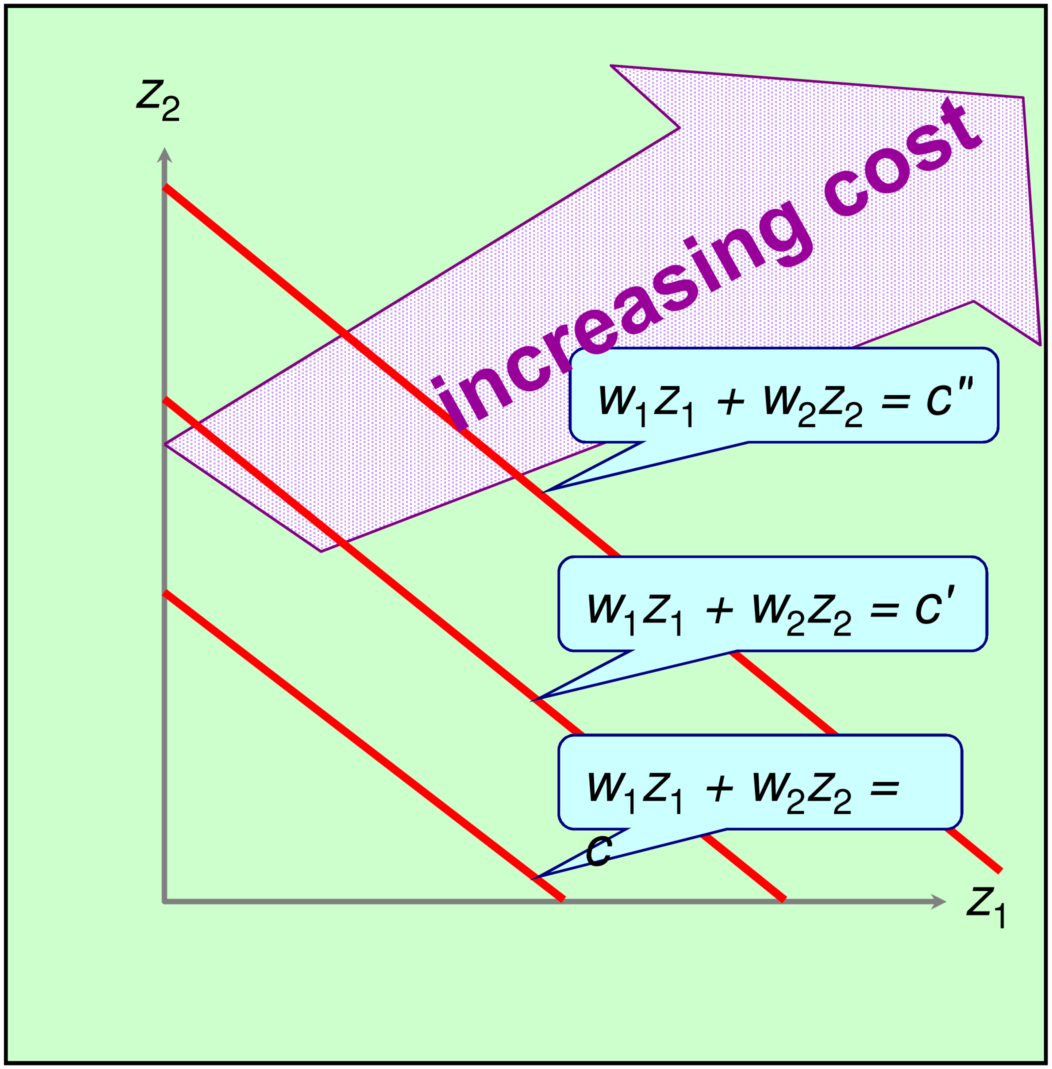
Optimisasi dengan batasan persamaan (Optimization with equality constraints)

- Mengapa batasan relevan dalam kajian ekonomi?
  - Masalah ekonomi timbul karena kelangkaan (scarcity).
  - Kelangkaan menyebabkan keputusan ekonomi (termasuk optimisasi) tidal dilakukan dalam kondisi tidak terbatas.
  - Dengan kata lain, constrained optimization merupakan pembahasan pokok dalam ekonomi

Lagrange Multiplier

- Merupakan suatu metode matematika yang dapat menyatakan suatu persoalan nilai ekstrim (maksimum atau minimum) yang mempunyai batasan (constrained-extremum) dalam bentuk yang bisa diselesaikan dengan menggunakan First-Order condition (FOC)

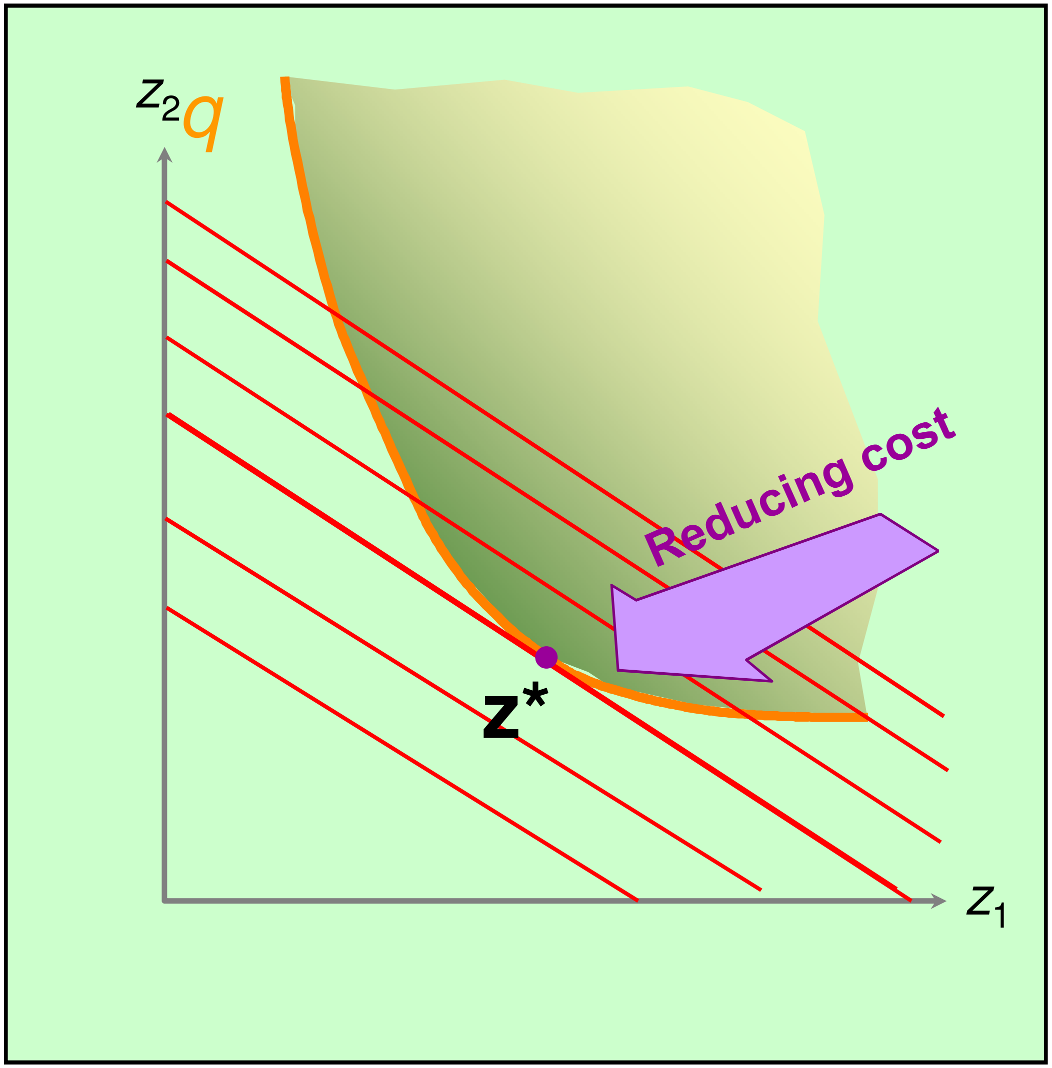
Iso-cost lines



- *Draw set of points where cost of input is c, a constant*
- *Repeat for a higher value of the constant*
- *Imposes direction on the diagram...*

Use this to derive optimum

Cost-minimisation



- *The firm minimises cost...*
- *Subject to output constraint*
- *Defines the stage 1 problem.*
- *Solution to the problem*

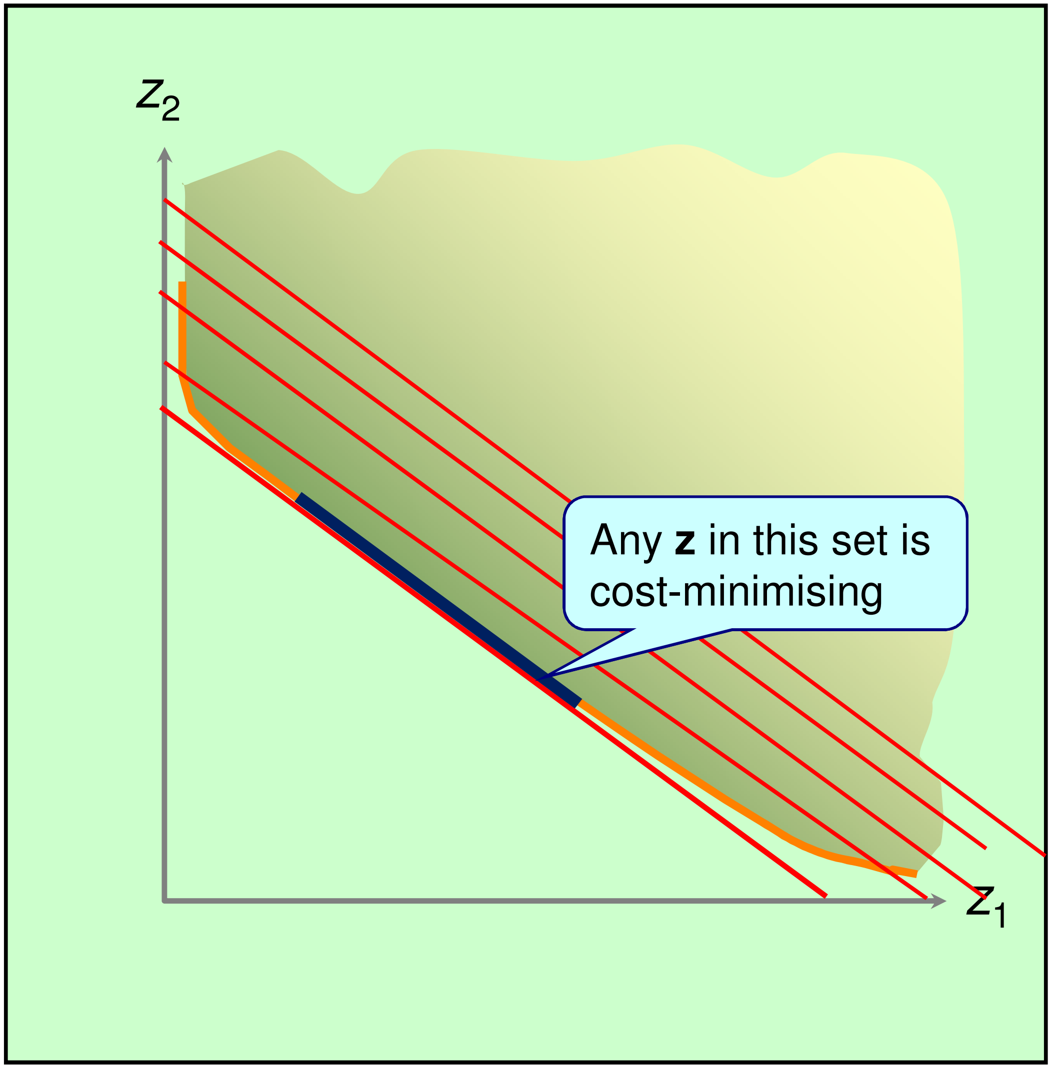
minimise

$$\sum_{i=1}^{m} w_i z_i$$

subject to $\phi(\mathbf{z}) \geq q$

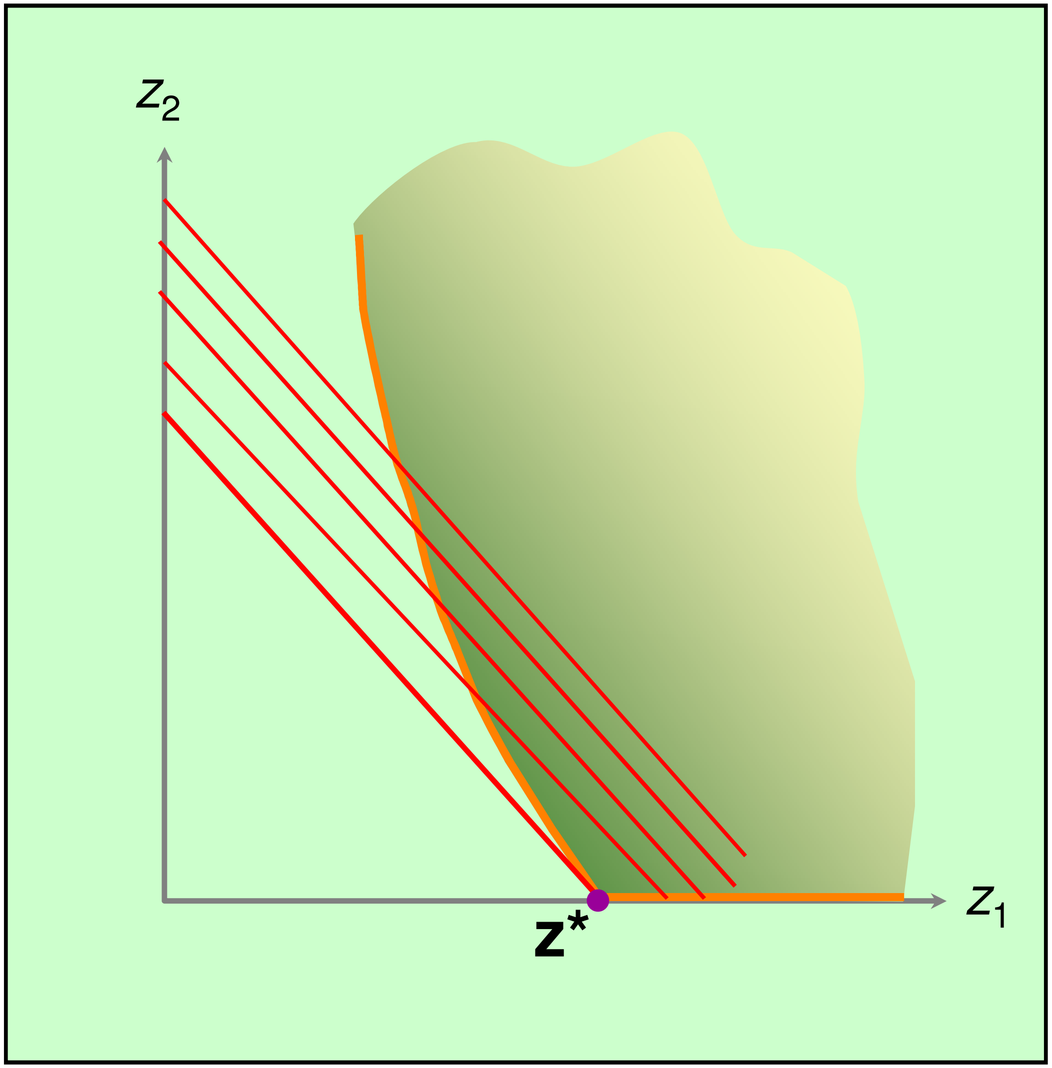
- *But the solution depends on the shape of the input-requirement set Z.*
- *What would happen in other cases?*

Convex, but not strictly convex *Z*



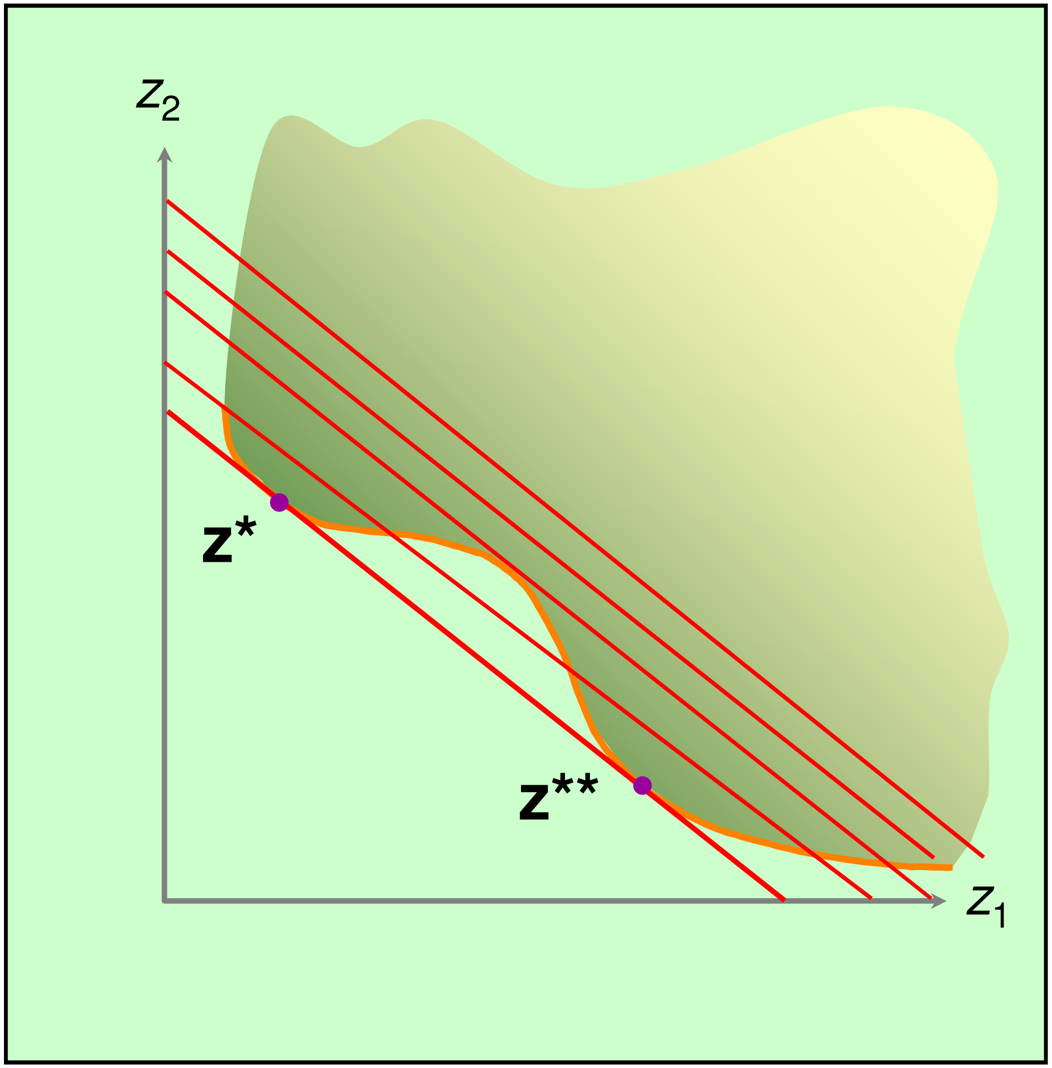
- *An interval of solutions*

Convex *Z*, touching axis



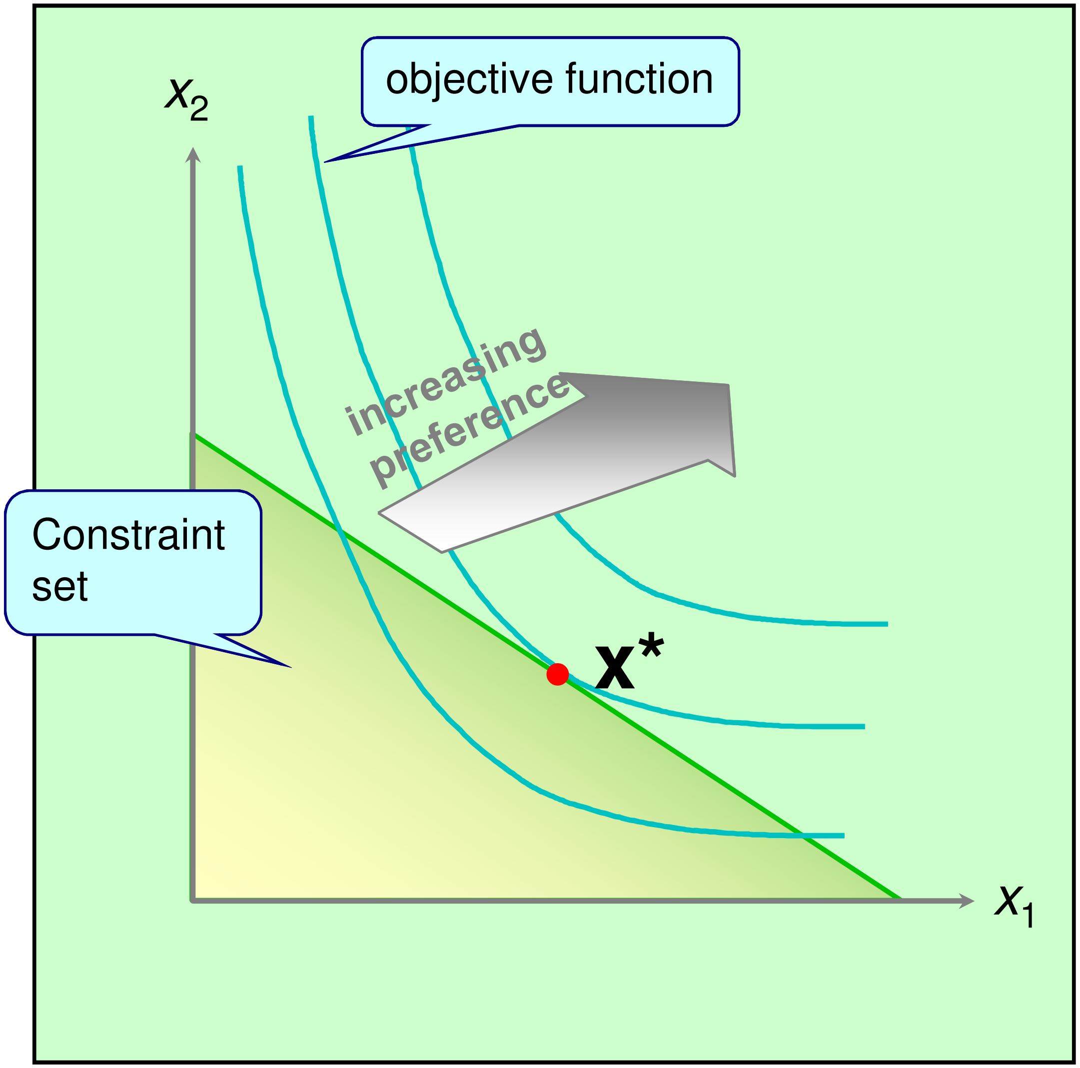
- *Here* $MRTS_{21} > w_1 / w_2$ *at the solution.*
- *Input 2 is "too expensive" and so isn't used:* $z_2{}^*=0$.

Non-convex Z



- *There could be multiple solutions.*
- *But note that there's no solution point between z* and z**.*

Aplikasi 1: Optimalisasi kepuasan konsumen
The primal problem



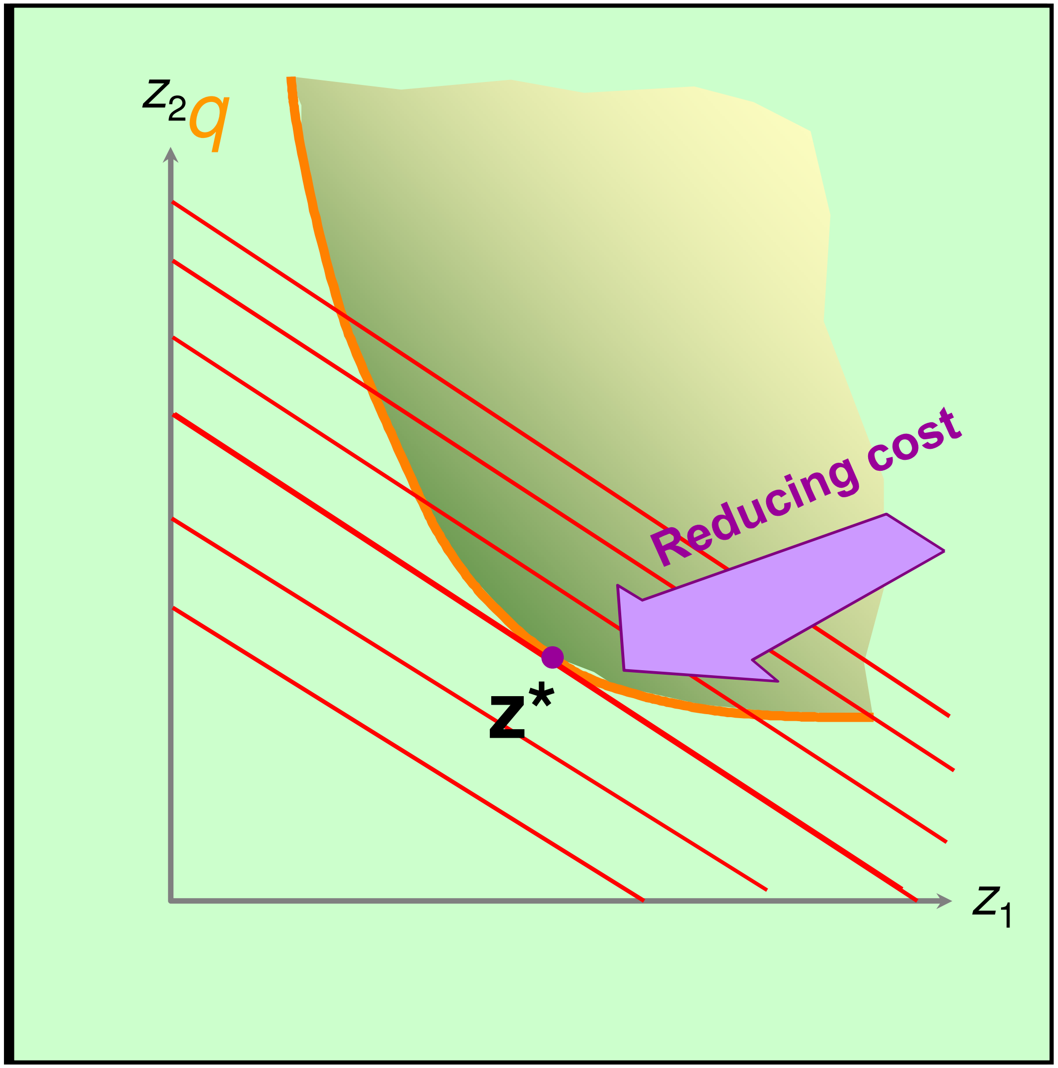
- ***Tujuan konsumen adalah memaksimalkan utilitas***
- ***Batasannya adalah budget***

max $U(\mathbf{x})$ subject to

$$\sum_{i=1}^{n} p_i x_i \leq y$$

- *Cara lain memandang persoalan ini adalah...*

The dual problem



- ***Konsumen bertujuan meminimalkan pengeluaran***
- ***Untuk mencapai utilitas tertentu***

minimise

$$\sum_{i=1}^{n} p_i x_i$$

subject to $U(\mathbf{x}) \geq \upsilon$

The Primal and the Dual…

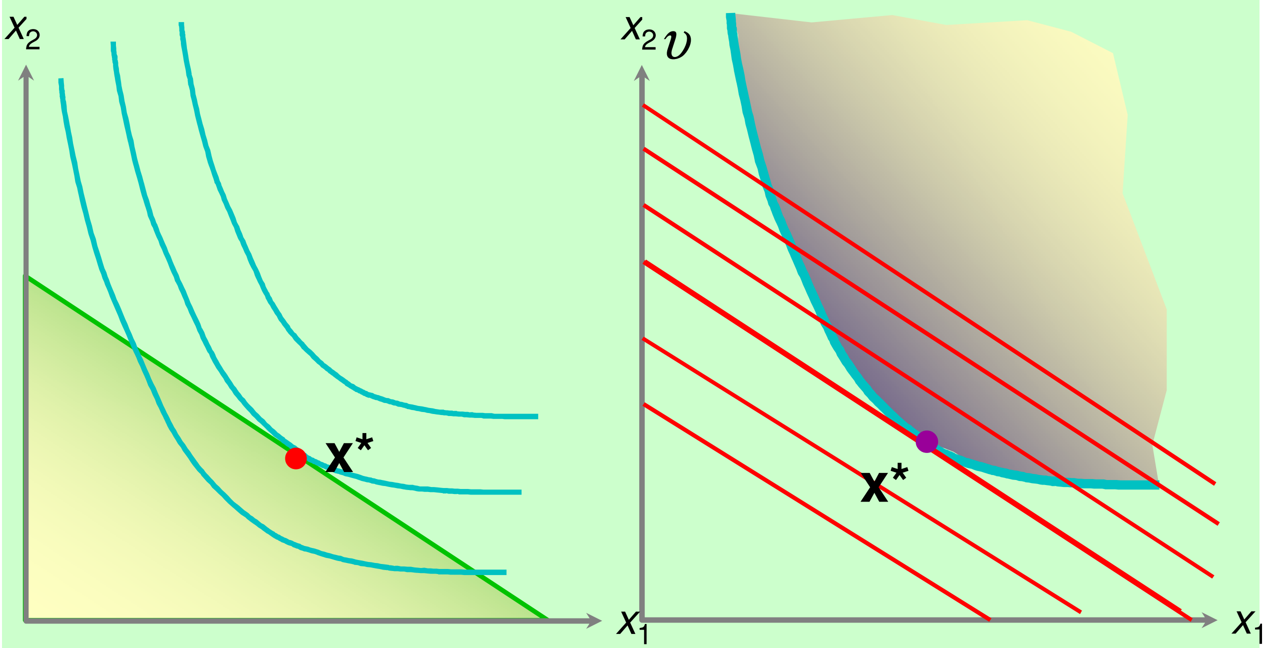
- There's an attractive symmetry about the two approaches to the problem
- In both cases the *p*s are given and you choose the *x*s. But…
- …constraint in the primal becomes objective in the dual…
- …and vice versa.

$$\sum_{i=1}^{n} p_i x_i + \lambda[\upsilon - U(\mathbf{x})]$$

$$U(\mathbf{x}) + \mu\left[y - \sum_{i=1}^{n} p_i x_i \right]$$

A neat connection

- *Compare the primal problem of the consumer...*
- *...with the dual problem*



- *The two are equivalent*
- *So we can link up their solution functions and response functions*

Run through the primal

Utilitas dan Pengeluaran

- Maksimisasi utilitas dan minimisasi pengeluaran pada dasarnya merupakan persoalan yang sama yang dilihat dari sudut pandang berbeda
- Dengan demikian, solusinya sangat terkait satu sama lainnya

	Primal	***Dual***
▪ Problem:	$\max_{\mathbf{x}} U(\mathbf{x}) + \mu\left[y - \sum_{i=1}^{n} p_i x_i\right]$	$\min_{\mathbf{x}} \sum_{i=1}^{n} p_i x_i + \lambda[\upsilon - U(\mathbf{x})]$
▪ Solution function:	$V(\mathbf{p}, y)$	$C(\mathbf{p}, \upsilon)$
▪ Response function:	$\boldsymbol{x}_i^* = D^i(\mathbf{p}, y)$	$\boldsymbol{x}_i^* = H^i(\mathbf{p}, \upsilon)$

Bentuk Umum

- Objective Function

$$z = f(x, y)$$

- Constraint

$$c = g(x, y)$$

- Lagrangian

$$L = f(x, y) + \lambda[c - g(x, y)]$$

Penyelesaian (FOC)

- Necessary Conditions

$$L_\lambda = c - g(x, y) = 0$$

$$L_x = f_x - \lambda g_x = 0$$

$$L_y = f_y - \lambda g_y = 0$$

Aplikasi 1: Maksimisasi utilitas dengan pendapatan terbatas

- Utility Function

$$U = x_1 x_2 + 2x_1$$

- Budget Constraint

$$4x_1 + 2x_2 = 60$$

- Lagrangian

$$L = x_1 x_2 + 2x_1 + \lambda[60 - 4x_1 - 2x_2]$$

Necessary Conditions

$$\frac{\partial L}{\partial \lambda} = 60 - 4x_1 - 2x_2 = 0$$

$$\frac{\partial L}{\partial x_1} = x_2 + 2 - 4\lambda = 0$$

$$\frac{\partial L}{\partial x_1} = x_1 - 2\lambda = 0$$

- Tentukan nilai x1 dan x2

Teorema Envelope

- Teorema yang membahas perubahan nilai optimal suatu fungsi dengan berubahnya salah satu parameter dalam fungsi tersebut

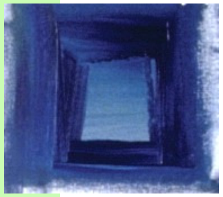
The Envelope Theorem

- Substituting into the original objective function yields an expression for the optimal value of y (y^*)

$$y^* = f[x_1^*(a), x_2^*(a), \ldots, x_n^*(a), a]$$

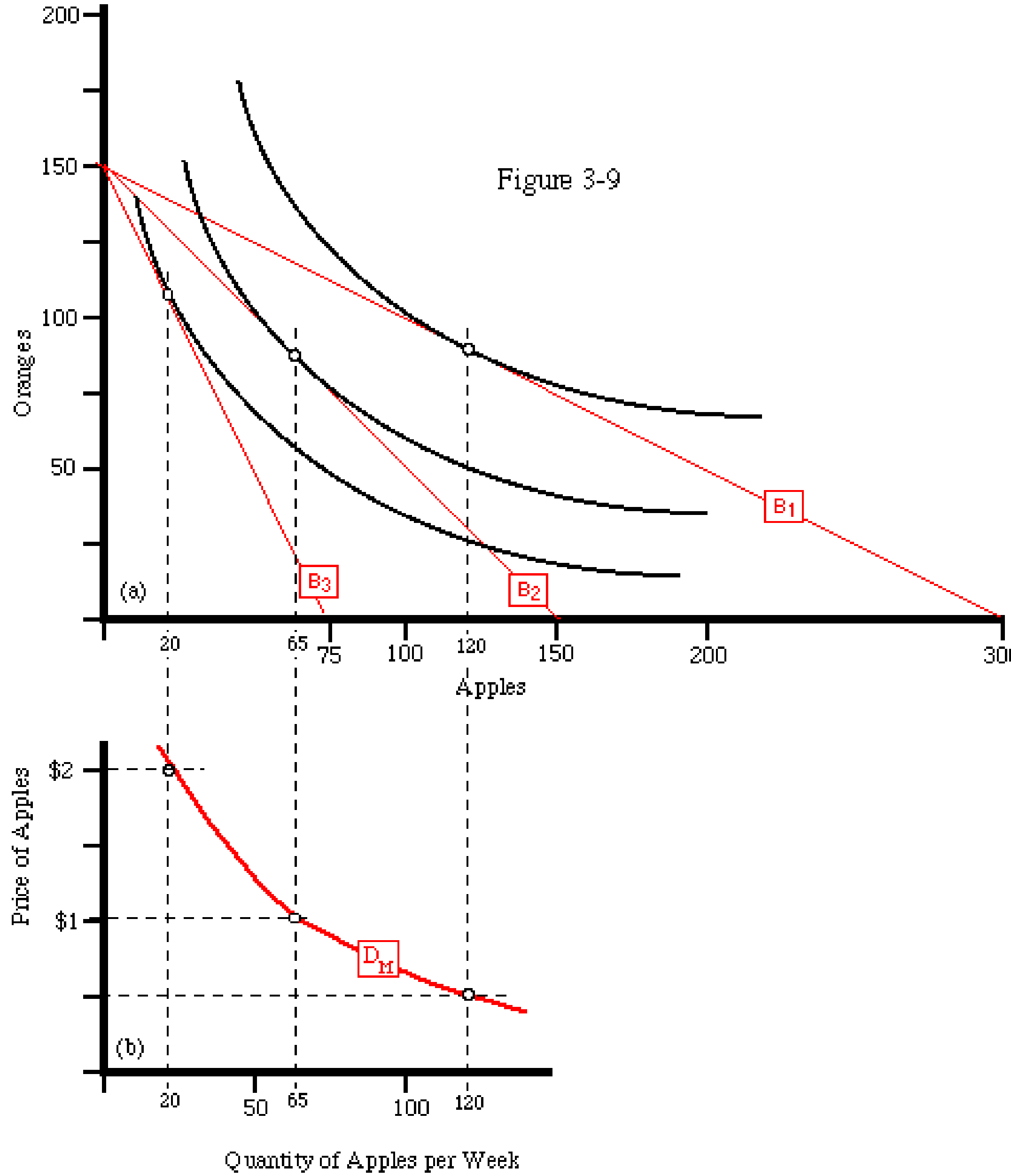
- Differentiating yields

$$\frac{dy^*}{da} = \frac{\partial f}{\partial x_1} \cdot \frac{dx_1}{da} + \frac{\partial f}{\partial x_2} \cdot \frac{dx_2}{da} + \ldots + \frac{\partial f}{\partial x_n} \cdot \frac{dx_n}{da} + \frac{\partial f}{\partial a}$$



Marshallian Demand

- The derivation of an ordinary demand curve. Budget lines B_1, B_2 and B_3 show different prices of apples but the same income and price of oranges. D_M is the ordinary (Marshallian) demand curve.



Hicksian Demand

- <u>The derivation of an income-adjusted demand curve.</u> Budget lines B_1, B_2 and B_3 show different combinations of prices and income corresponding to the same real income. D_H is the resulting income-adjusted (Hicksian) demand curve.