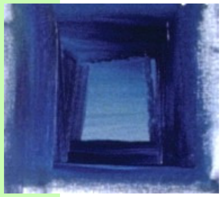


Trinity
MATRIX
get up
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Maksimisasi utilitas dengan pendapatan terbatas

- Utility Function

$$U = x_1 x_2 + 2x_1$$

- Budget Constraint

$$4x_1 + 2x_2 = 60$$

- Lagrangian

$$L = x_1 x_2 + 2x_1 + \lambda[60 - 4x_1 - 2x_2]$$

- Necessary Conditions

$$\frac{\partial L}{\partial \lambda} = 60 - 4x_1 - 2x_2 = 0$$

$$\frac{\partial L}{\partial x_1} = x_2 + 2 - 4\lambda = 0$$

$$\frac{\partial L}{\partial x_1} = x_1 - 2\lambda = 0$$

- Tentukan nilai x1 dan x2

Bentuk Umum

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = d_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = d_2$$

....

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = d_m$$

Ukuran matriks

- Matrix [A] akan disebut berukuran mxn jika mempunyai m baris dan n kolom
- Lambangnya adalah $[A]_{mxn}$
- Elemennya disimbolkan dengan a_{ij}, dimana i merupakan urutan baris dan j urutan kolom.

Matriks

$$Ax = d$$

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad d = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_m \end{bmatrix}$$

$$A_{mxn} = \left[a_{ij} \right] \qquad \begin{pmatrix} i = 1, 2, \ldots, m \\ j = 1, 2, \ldots, n \end{pmatrix}$$

Solusi

$$Ax = d$$

$$x = A^{-1}d$$

Perkalian Matriks

$$A_{mxn} B_{nxo} = C_{mxo}$$

syarat [m x n] dan [n x o]

hasil [m x o]

Contoh

$$A = \begin{bmatrix} 1\,3 \\ 2\,8 \\ 4\,0 \end{bmatrix} \qquad b = \begin{bmatrix} 5 \\ 9 \end{bmatrix}$$

$$C = \begin{bmatrix} 1(5)+3(9) \\ 2(5)+8(9) \\ 4(5)+0(9) \end{bmatrix} = \begin{bmatrix} 32 \\ 82 \\ 20 \end{bmatrix}$$

Contoh

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

[3 x 2] [2 x 3]

A and B can be multiplied

$$C = \begin{bmatrix} 2*1+3*1=5 & 2*1+3*0=2 & 2*1+3*2=8 \\ 1*1+1*1=2 & 1*1+1*0=1 & 1*1+1*2=3 \\ 1*1+0*1=1 & 1*1+0*0=1 & 1*1+0*2=1 \end{bmatrix} = \begin{bmatrix} 5\,2\,8 \\ 2\,1\,3 \\ 1\,1\,1 \end{bmatrix}$$

[3 x 3]

Contoh

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

[3 x 2] [2 x 3]

Result is 3 x 3

$$C = \begin{bmatrix} 2*1+3*1=5 & 2*1+3*0=2 & 2*1+3*2=8 \\ 1*1+1*1=2 & 1*1+1*0=1 & 1*1+1*2=3 \\ 1*1+0*1=1 & 1*1+0*0=1 & 1*1+0*2=1 \end{bmatrix} = \begin{bmatrix} 5\,2\,8 \\ 2\,1\,3 \\ 1\,1\,1 \end{bmatrix}$$

[3 x 3]

Operasi matriks

- Perkalian skalar

$$7\begin{pmatrix} 3 & -1 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} 21 & -7 \\ 0 & 35 \end{pmatrix}$$

$$c\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} ca_{11} & ca_{12} \\ ca_{21} & ca_{22} \end{pmatrix}$$

Keunikan matriks

$$AB \neq BA$$

$$A = \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$$

$$\frac{\partial L}{\partial \lambda} = 60 - 4x_1 - 2x_2 = 0$$

$$\frac{\partial L}{\partial x_1} = x_2 + 2 - 4\lambda = 0$$

$$\frac{\partial L}{\partial x_1} = x_1 - 2\lambda = 0$$

⇒

$$-4x_1 - 2x_2 + 0\lambda = -60$$

$$0x_1 + x_2 - 4\lambda = -2$$

$$x_1 + 0x_2 - 2\lambda = 0$$

Matriks

$$Ax = d$$

$$\begin{pmatrix} -4 & -2 & 0 \\ 0 & 1 & -4 \\ 1 & 0 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \lambda \end{pmatrix} = \begin{pmatrix} -60 \\ -2 \\ 0 \end{pmatrix}$$

Operasi matriks

$$A_{mxn}B_{nxo} = C_{mxo}$$

$$\begin{pmatrix} -4 & -2 & 0 \\ 0 & 1 & -4 \\ 1 & 0 & -2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ \lambda \end{pmatrix} = \begin{pmatrix} -60 \\ -2 \\ 0 \end{pmatrix}$$

$$A = 3x3$$

$$B = 3x1$$

$$C = 3x1$$

Solusi

$$x = A^{-1}d$$

$$\begin{pmatrix} x_1 \\ x_2 \\ \lambda \end{pmatrix} = \begin{pmatrix} -4 & -2 & 0 \\ 0 & 1 & -4 \\ 1 & 0 & -2 \end{pmatrix}^{-1} \begin{pmatrix} -60 \\ -2 \\ 0 \end{pmatrix}$$

Inverse

$$A^{-1} = \frac{1}{\det .A} adj.A$$

Determinan

- Hanya matriks yang bujursangkar (baris = kolom) yang mempunyai determinan

Matriks 2x2

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$\det A = a_{11}a_{22} - a_{12}a_{21}$$

Matriks 3x3

Langkah 1: Tambahkan 2 kolom yang pertama

$$\begin{vmatrix} -4 & -2 & 0 \\ 0 & 1 & -4 \\ 1 & 0 & -2 \end{vmatrix} \Rightarrow \left|\begin{array}{ccc|cc} -4 & -2 & 0 & -4 & -2 \\ 0 & 1 & -4 & 0 & 1 \\ 1 & 0 & -2 & 1 & 0 \end{array}\right.$$

Matriks 3x3

Langkah 2: Jumlahkan perkalian diagonal

$$\left|\begin{array}{ccc|cc} -4 & -2 & 0 & -4 & -2 \\ 0 & 1 & -4 & 0 & 1 \\ 1 & 0 & -2 & 1 & 0 \end{array}\right.$$

[(-4x1x-2)+(-2x-4x1)+(0x0x0)]-[(1x1x0)+(0x-4x-4)+(-2x0x-2)]

Minors of a Matrix Determinant

- A minor $M_{i,j}$ is a reduced determinant found by omitting the i^{th} row and j^{th} column of a larger determinant. For example:

Let:

$$\det(\mathbf{A}) = \begin{vmatrix} A_{1,1} & A_{1,2} & A_{1,3} & \dots & A_{1,n} \\ A_{2,1} & A_{2,2} & A_{2,3} & \dots & A_{2,n} \\ A_{3,1} & A_{3,2} & A_{3,3} & \dots & A_{3,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{n,1} & A_{n,2} & A_{n,3} & \dots & A_{n,n} \end{vmatrix}$$

Then

$$\mathrm{M}_{2,2} = \begin{vmatrix} A_{1,1} & A_{1,3} & \dots & A_{1,n} \\ A_{3,1} & A_{3,3} & \dots & A_{3,n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n,1} & A_{n,3} & \dots & A_{n,n} \end{vmatrix}$$

The Cofactor of Determinants

- A *cofactor* $\mathbf{C}_{i,j}$ is a minor $M_{i,j}$ augmented with a sign rule for the particular purpose of solving matrix determinants. Cofactors are defined as follows:

A cofactor $\mathrm{C}_{i,j}$ involves the minor $\mathrm{M}_{i,j}$ such that:

$$\mathrm{C}_{i,j} = (-1)^{i+j}\mathrm{M}_{i,j}$$

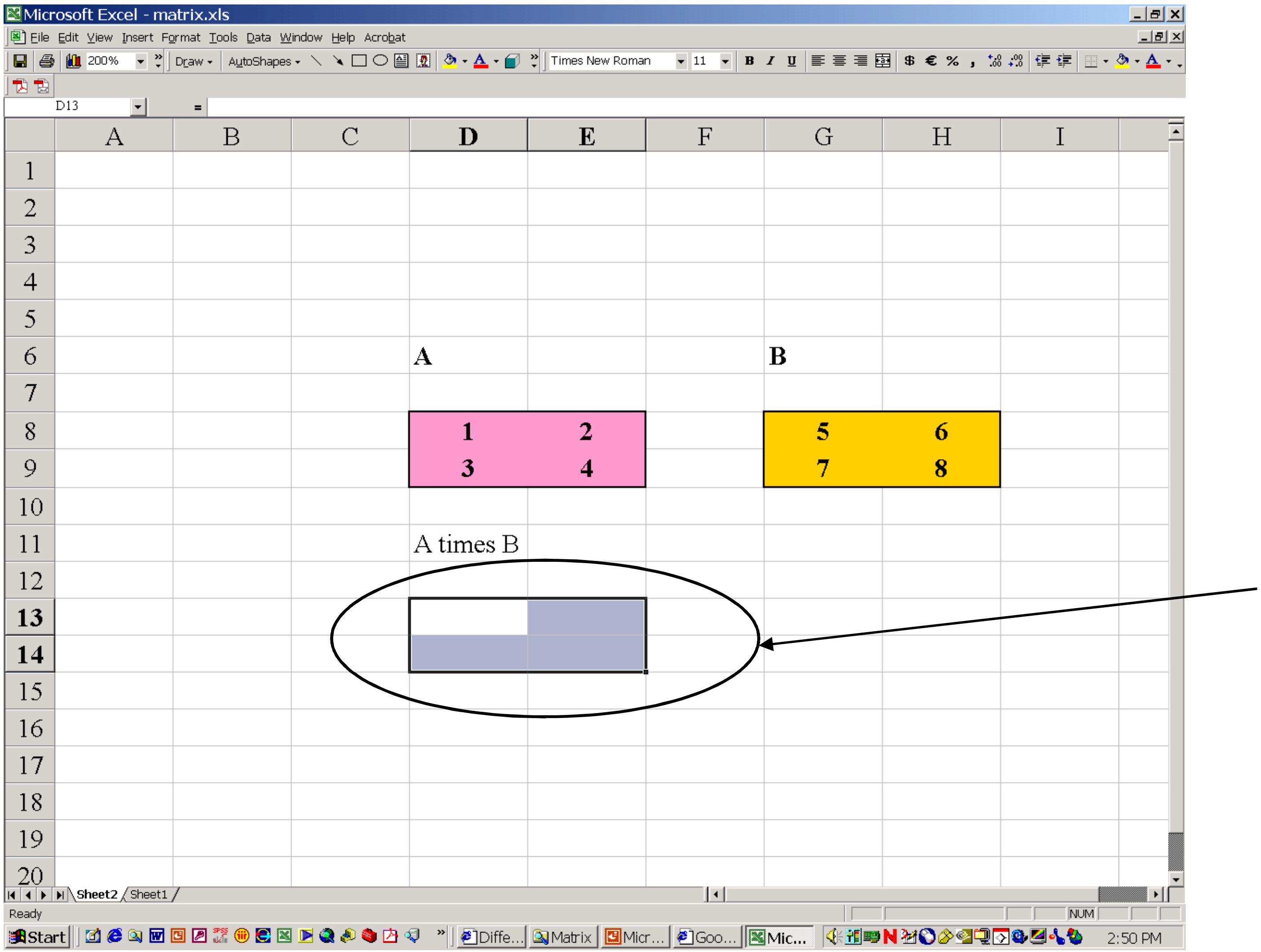
$$A = \begin{vmatrix} -4 & -2 & 0 \\ 0 & 1 & -4 \\ 1 & 0 & -2 \end{vmatrix}$$

$$C_{11} = (-1)^{1+1} \begin{vmatrix} -4 & -2 & 0 \\ 0 & 1 & -4 \\ 1 & 0 & -2 \end{vmatrix} \Rightarrow C_{11} = \begin{vmatrix} 1 & -4 \\ 0 & -2 \end{vmatrix}$$

Adjoint

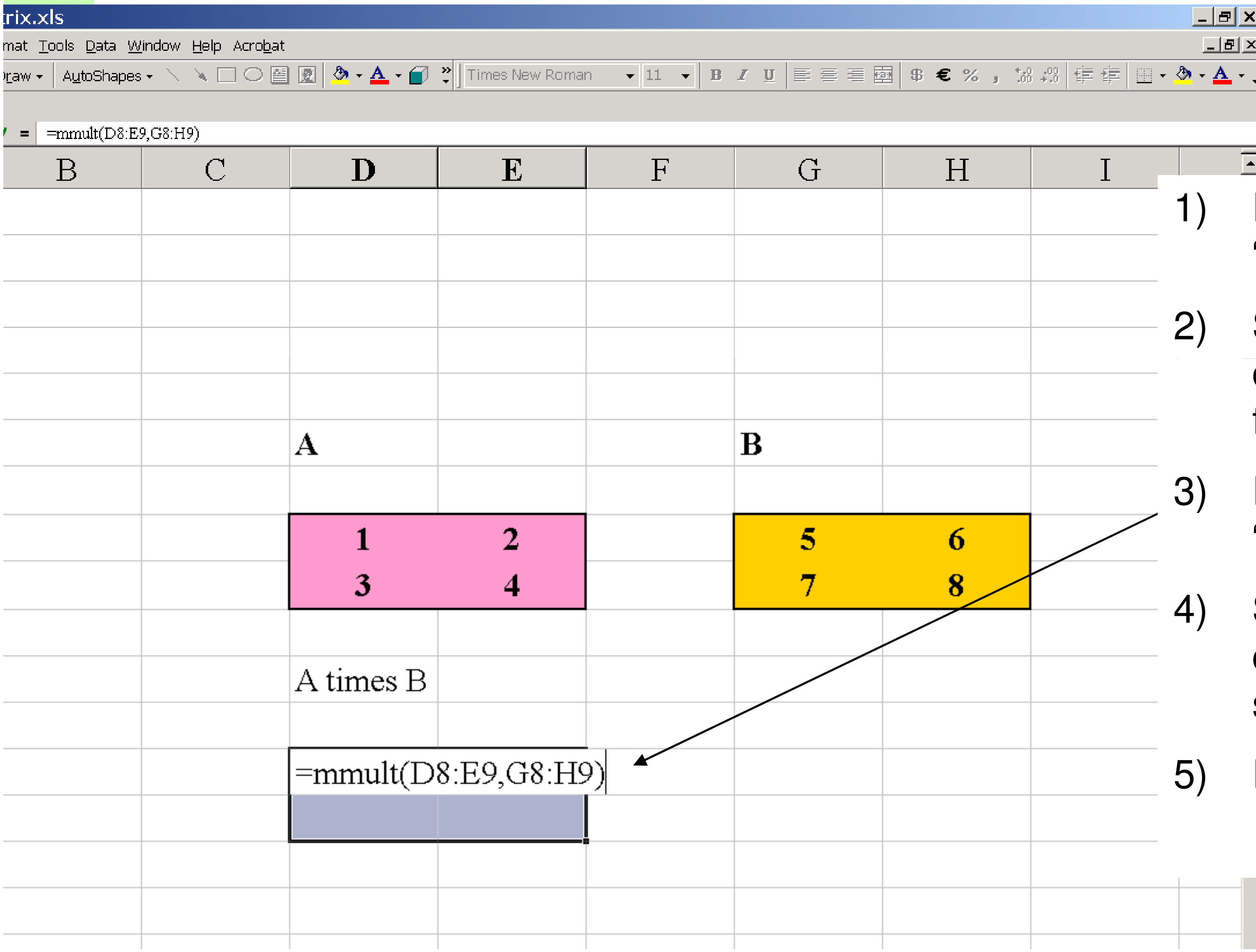
$$adj.A = \begin{pmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{pmatrix}$$

Matrix Operations in Excel

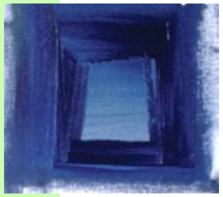


Select the cells in which the answer will appear

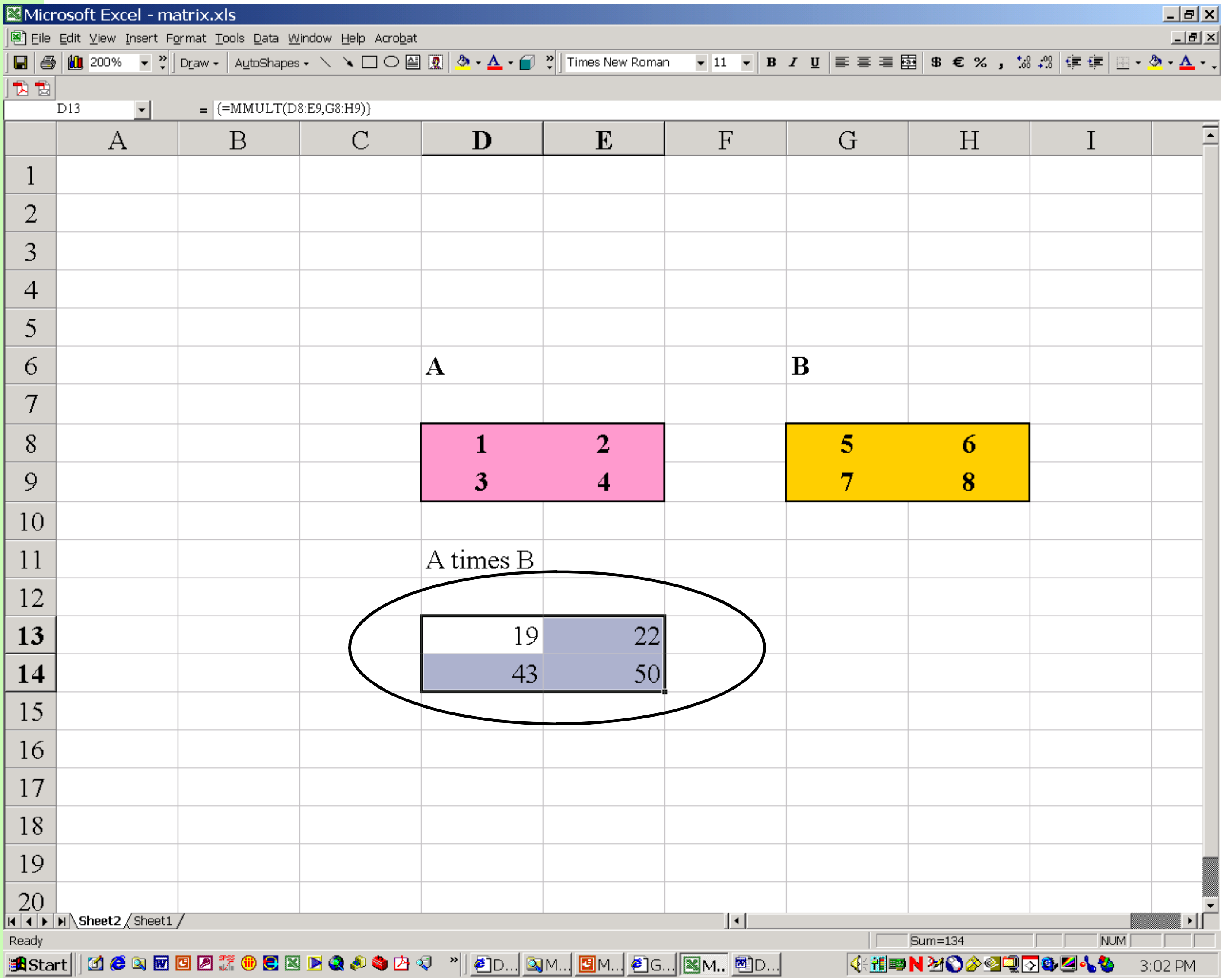
Matrix Multiplication in Excel



1) Enter "=mmult("
2) Select the cells of the first matrix
3) Enter comma ","
4) Select the cells of the second matrix
5) Enter ")"



Matrix Multiplication in Excel



Enter these three key strokes at the same time:

control

shift

enter

Matrix Inversion in Excel

- Follow the same procedure
- Select cells in which answer is to be displayed
- Enter the formula: =minverse(
- Select the cells containing the matrix to be inverted
- Close parenthesis – type ")"
- Press three keys: Control, shift, enter

- matrix.xlsx